Dark Matter as a Possible New Energy Source for Future Rocket Technology

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Current rocket technology can not send the spaceship very far, because the amount of the chemical fuel it can take is limited. We try to use dark matter (DM) as fuel to solve this problem. In this work, we give an example of DM engine using dark matter annihilation products as propulsion. The acceleration is proportional to the velocity, which makes the velocity increase exponentially with time in non-relativistic region. The important points for the acceleration are how dense is the DM density and how large is the saturation region. The parameters of the spaceship may also have great influence on the results. We show that the (sub)halos can accelerate the spaceship to velocity $10^{-5}c \sim 10^{-3}c$. Moreover, in case there is a central black hole in the halo, like the galactic center, the radius of the dense spike can be large enough to accelerate the spaceship close to the speed of light.

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I. INTRODUCTION

It is difficult for human to reach the stars using current rocket technology. The energy sources range from chemical fuel, nuclear power and even anti-matter conceptually. The major problem in these systems is that propulsion required large amount of time and fuel[1]. We try to solve this problem by starting at the requirement of large amount of fuel. We all know that current rockets in function are chemical rockets which take oxidant and fuel at the same time. Interestingly, the airplanes with similar propulsion system only take fuel, without oxidant, because in the atmosphere there are enough oxygen which are absorbed by airplane engines during the flight. Similarly, if there are enough fuel in the universe, the spaceship may absorb them during its flight like airplanes absorb the oxygen. Fortunately, dark matter is widely spread in the universe and the mass density is about five times of the baryonic matter density[2], which make it a possible new energy source for interstellar flight. Thus the requirement of fuel may be solved in the self-help way with dark matter as the energy source.

II. DM ENGINE AND ACCELERATION IN THE SATURATION DENSITY

We give an example of DM engine which uses DM annihilation remnants as propulsion. Fig.1 is a sketch of the DM engine for this kind of new spaceship. The DM engine is the box in the picture. Here we assume the DM particle and the annihilation products can not pass through the wall of the box. In picture A, the space ship moves very fast from right to left. The DM particles, which are assumed to be static, go into the box and are absorbed in the picture B. In the picture C, we compress the box and raise the number density of the DM for annihilation, where we assume the annihilation process happens immediately. In the picture D, only the wall on the right side is open. The annihilation products, for example Standard Model (SM) particles, are all going to the right direction. The processes from A to D are the working cycle for the engine. Thus, the spaceship is boosted by the recoil of these SM particles. Note the spaceship can decelerate by the same system when it reaches the destination, by opening the left wall in the picture D.

This kind of new spaceship has a very interesting character that the faster it is, the easier it accelerates. In the picture A, we assume the rest mass of the spaceship is M and its velocity is β in the unit of speed of light. The time for one cycle of the engine is dt and the area of the engine is S. During one work cycle, the number of DM particles collected by the engine is $N = \beta dt \cdot S \cdot \frac{\rho_D}{m_D}$, where ρ_D and m_D are the density of DM and mass of DM, respectively. In picture D, we assume there are only one kind of particles X as the annihilation products for simplicity. The annihilation process is $DD \to X\bar{X}$, with the mass $m_X < m_D$. For the DM particles, we assume DM mass $m_D \sim O(100GeV)$ and

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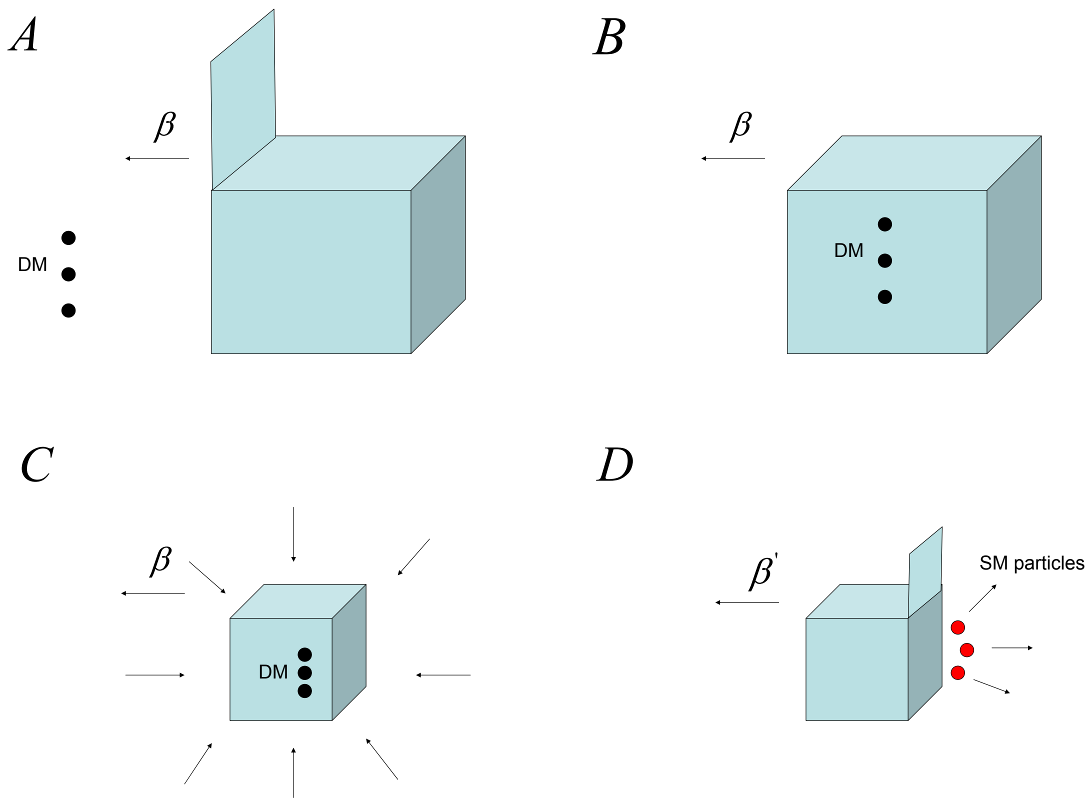

FIG. 1: The illustration of work cycle for the DM engine.

the annihilation products to be SM fermions mainly, which are quite natural in SuperSymmetry and Extra Dimension models. Thus, the mass of the annihilation products m_X are quite small comparing with the mass of dark matter m_D. So it is reasonable to use the approximation $m_X = 0$, where products are treated as massless photon in the following calculation. Using the conservation of energy and momentum, we can get

$$\begin{cases} \beta dt \cdot S \cdot \rho_D + p^0 = p^0 + dp^0 + \varepsilon \\ 0 + p^1 = p^1 + dp^1 - \varepsilon \cdot \theta \end{cases} \tag{1}$$

where $p^0 = M\gamma$ and $p^1 = M\gamma\beta$ are the energy and the momentum of the spaceship, and $\gamma \equiv (1-\beta^2)^{-1/2}$, ε is the energy of the massless photons. θ is defined as the propulsion efficiency, which means $\theta \in [0, 1]$. For example, if the annihilation particles all go to the right direction, then $\theta = 1$. However, if the annihilation particles have equal possibility go into any direction in the right hemisphere, then $\theta = 1/2$. Moreover, it can also be used to count in the other inefficiency of the engine. By the Eqn.1, one can get the differential equation for the velocity,

$$\frac{k\beta}{\theta^{-1} + \beta} = \gamma^3 \frac{d\beta}{dt} \tag{2}$$

where $k \equiv S\rho_D/M$. In the non-relativistic region, the above equation has the simple form,

$$\theta k \cdot \beta = \frac{d\beta}{dt}. \tag{3}$$

To carry out the numerical calculations, we give some reasonable parameters first. We assume the weight of the spaceship is $M = 100ton$ and the area is $S = 100m^2$, according to the current rockets and space shuttles. For the DM density, we use the saturation density in the center of cusped halos ρ_{sat}. The saturation density in the halo is due to the balance between the annihilation rate of the DM $[\langle \sigma v \rangle \, \rho_{sat}/m_D]^{-1}$ and the gravitational in falling rate of the DM $(G\bar{\rho})^{-1/2}$, where the $\bar{\rho}$ is taken to be 200 times of the critical density. Thus the saturation density is $\rho_{sat} \sim 10^{19} M_{\odot} \cdot kpc^{-3}$[3, 4]. The propulsion efficiency is taken to be $\theta = 0.5$, since in the picture D we assume the annihilation particles have equal possibility go into any direction in the right hemisphere. We can also rewrite the parameter k as following,

$$k = 2 \times 10^{-4} s^{-1} \cdot \left(\frac{\rho}{10^{19} M_{\odot} \cdot kpc^{-3}} \frac{S}{100m^2} \frac{100ton}{M} \right) \tag{4}$$

One can solve the Eqn.2 and get the time and length needed for acceleration as the function of velocity,

$$t = (\theta k)^{-1} \cdot [\frac{1 + \theta\beta}{\sqrt{1-\beta^2}} + In(\frac{\beta}{1+\sqrt{1-\beta^2}})] \Big|_{\beta_0}^{\beta}, \tag{5}$$

$$L = (\theta k)^{-1} \cdot \frac{\beta + \theta}{\sqrt{1-\beta^2}} \Big|_{\beta_0}^{\beta}. \tag{6}$$

where β_0 is the initial velocity at the $t = 0$. We give the plots of the above equations in Fig.2. We can see the velocity increases exponentially with time, since the acceleration is proportional to the velocity. In the non-relativistic region, where $\beta \ll 1$, the Eqn.5 and the Eqn.6 can be simplified as

$$\beta = \beta_0 e^{\theta k t}, \tag{7}$$

$$L = (\theta k)^{-1} (\beta - \beta_0). \tag{8}$$

The initial velocity β_0 is taken to be $10^{-6} c$ which is much smaller than the first cosmic velocity. However, the result is not sensitive to the initial velocity, because of the exponential increase. In Fig.2, we see the spaceship can reach the relativistic speed in about 2 *days* and the length needed for acceleration is about $10^{-4} pc$. From the above equations, if the DM density ρ and the area of the spaceship S are larger, the time t and length L needed for acceleration will go down. If the mass of spaceship M is larger, the time t and length L needed for acceleration will increase. However, the mass of DM particle does not have great influence on results, but the DM density does.

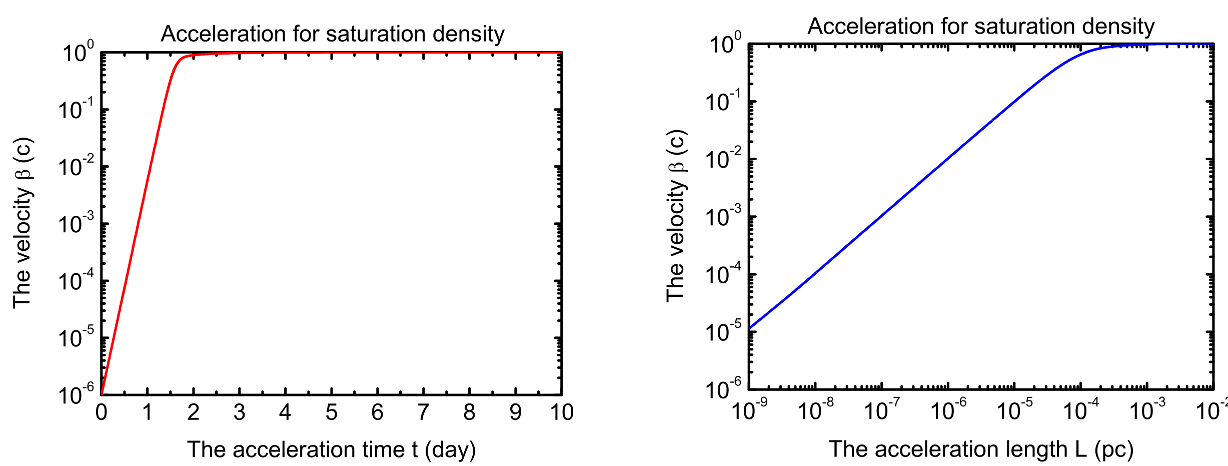

FIG. 2: The velocity as a function of the time and length needed for the acceleration in the saturation density.

III. ACCELERATION IN THE HALO OR SUBHALO

Before celebration for the reach of relativistic speed, we should check whether the saturation region in the halo or subhalo is large enough for the above calculation. The DM profile can be parameterized as $\rho(r) = \frac{\rho_s}{(r/r_s)^\gamma[1+(r/r_s)^\alpha]^{(\beta-\gamma)/\alpha}}$, where ρ_s and r_s are the scale density and scale radius parameters respectively. The parameters (α, β, γ) are $(1, 3, 1)$ for NFW profile[5]. Since we are interested in the central region of halo, where $r \ll r_s$, the profile can be simplified as,

$$\rho = \frac{\rho_s r_s}{r}. \tag{9}$$

This profile is singular at the center of the halo. It is natural to have cut-off for this singularity due to the balance between the annihilation rate of the DM and the gravitational in falling rate of the DM. The saturation DM density is taken to be $\rho_{sat} \sim 10^{19} M_\odot \cdot kpc^{-3}$, thus the radius of saturation is $r_{sat} = \rho_s r_s/\rho_{sat}$. Once we know the scale density ρ_s and scale radius r_s, we can calculate the saturation radius r_{sat}. The ρ_s and r_s can be fully determined by the concentration model and DM halo mass, which will be calculated in the appendix. Here we show the saturation radius r_{sat} in Fig.3. We can see the saturation radius of halo or subhalo is much smaller than the required length for acceleration to the relativistic speed.

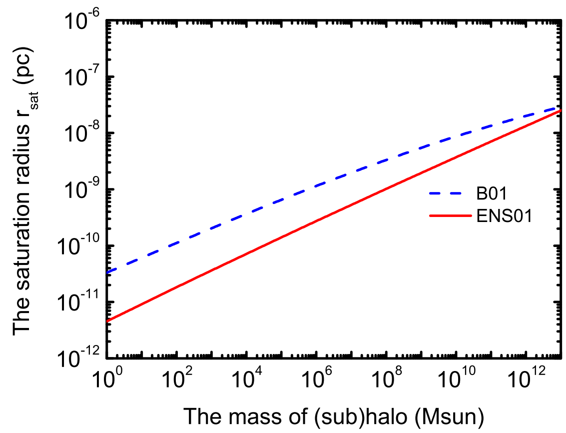

FIG. 3: The saturation radius r_{sat} for different (sub)halo mass and concentration models. The B01 and ENS01 stand for different concentration models which are described in the appendix.

In Fig.4, we show the details of acceleration in the subhalo. The subhalo with mass $10^6 M_\odot$ in B01 model is taken as an example, which has saturation radius of about $10^{-9}pc$. Starting from the center of subhalo with initial velocity $\beta_0 = 10^{-6}c$, it reaches the velocity of about $10^{-5}c$ when it leaves the saturation region, which can be read out from the Fig.2. However, the rest of the subhalo is not sufficient to accelerate the spaceship to the relativistic speed, since the density begins to decrease by r^{-1}. By solving the differential equation numerically, we can get the relations among velocity, time and distance in Fig.4. We can see the spaceship reaches the velocity $10^{-4}c$ in about ten days. However, its velocity hardly increases after that, since the DM density goes down quickly. We can see that the acceleration is fastest in the saturation area of the halo. But the rest region of subhalo can still accelerate the spaceship from the velocity $10^{-5}c$ to the velocity $10^{-4}c$.

To better understand the acceleration power of the (sub)halos, we give the velocity at different times for different (sub)halos mass and spaceship parameters in Fig.5. From the picture on the left, we can find that the subhalos have the power to accelerate the spaceship to velocity $10^{-5}c \sim 10^{-3}c$ with reasonable parameters $S/M = 100m^2/100ton$. In the first few hours, the spaceship flies in the saturation region where they will be accelerated to velocity $10^{-6}c \sim 10^{-4}c$, which can be understood with the help of Fig.2 and Fig.3. Out of the saturation region, the velocity of the spaceship can get further boosted by about one order in the r^{-1} density region. Note that the above accelerations take place at the very center of halo, which is far less than the scale radius r_s. The above results rely on the parameters of spaceship, e.g. the ratio S/M. If we lower the the weight of spaceship and increase the area of the engine, the velocity we can achieve will significantly increase. We specially give the plot on the right for S/M which is ten times larger, although the parameters maybe unreasonable in practice. It shows the the corresponding velocity increases about ten times. The main reason is the velocity at $r = r_{sat}$ increases by ten times, which can be understood with the Eqn.8.

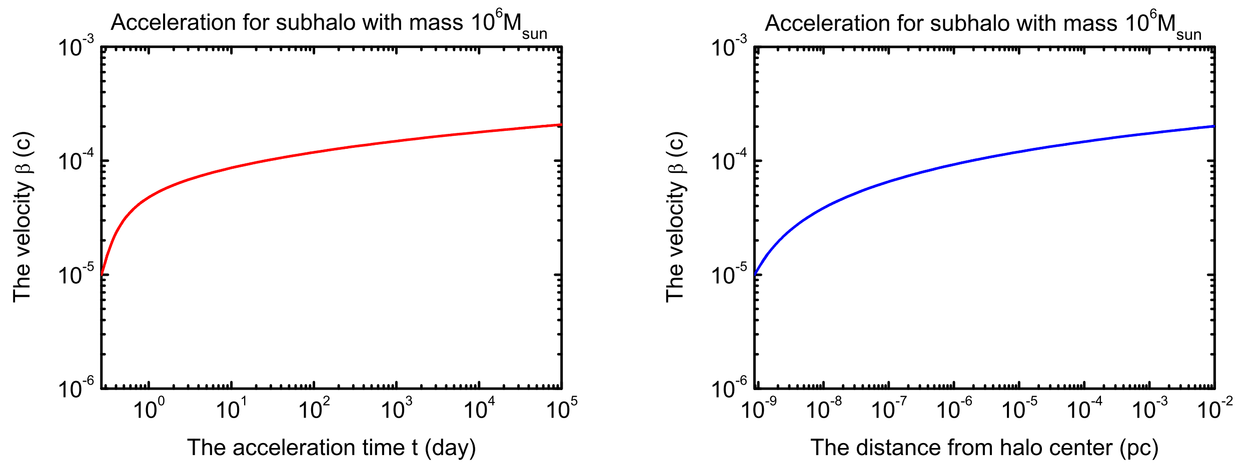

FIG. 4: The velocity as a function of the time and length needed for the acceleration in the DM (sub)halo. The spaceship starts from the (sub)halo center with initial velocity $\beta_0 = 10^{-6}c$.

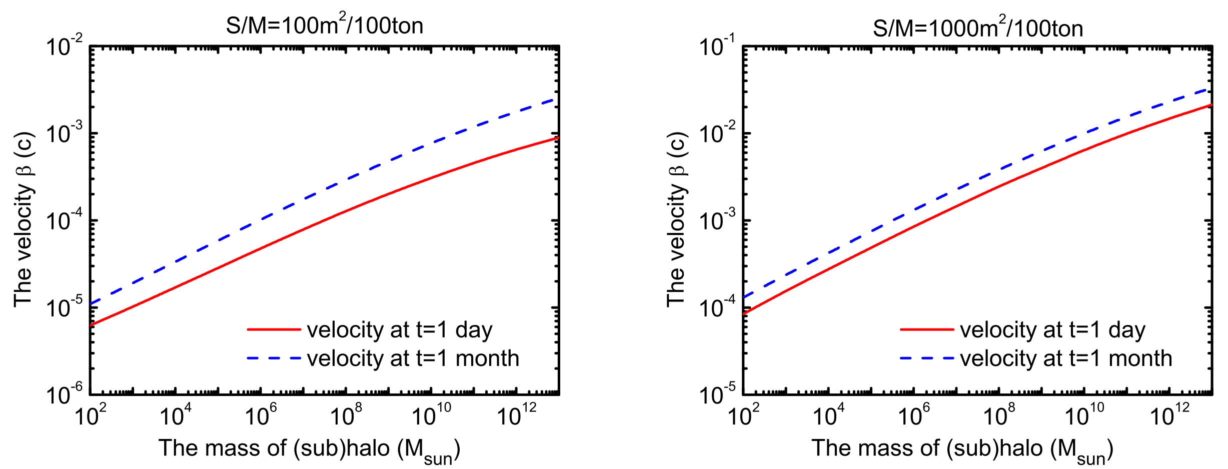

FIG. 5: The velocity at time $t = 1day$ and $t = 1month$ for different (sub)halos mass and spaceship parameters. The concentration model is taken to be B01. The spaceship is still assumed to be started at the center of (sub)halo with initial velocity $\beta_0 = 10^{-6}c$.

Anyway, the (sub)halos seem difficult to boost the spaceship to relativistic velocity, because their saturation radius is small comparing with the required acceleration length $10^{-4}pc$ in the Fig.2. In the above calculation, we assume there are no baryonic matter in the halo. The gravity from the DM halo have negligible effects on the spaceship, even at the saturation region. Note the saturation density $\rho_{sat} \sim 10^{19} M_\odot \cdot kpc^{-3}$ is much smaller than the density of water $1g/cm^3 \sim 10^{31} M_\odot \cdot kpc^{-3}$.

However, in case there are baryonic matter in the halo, it may modify the DM profile. The adiabatic contraction due to dissipating baryons can steepen the DM profile[6]. A more interesting case is that there is a central black hole in the DM halo, for example at galactic center. The DM density can become a dense spike due to accretion by the black hole, assuming adiabatic growth of the black hole[7]. The annihilations in the inner regions of the spike set a maximal dark matter density $\rho_{core} = \frac{m_D}{\langle \sigma v \rangle t_{bh}} \sim 10^{17} M_\odot \cdot kpc^{-3}$, where m_D is the mass of DM particle, and t_{bh} is the age of black hole, conservatively $10^{10}yr$. And more importantly, the radius of the core can be as large as $O(10^{-2}pc)$ for inner cusped model like NFW profile. Recall the Eqn.6, the required acceleration length for velocity $0.9c$ is about $O(10^{-2}pc)$ in this case, which means the spaceship can achieve the velocity close to the speed of light.

IV. CONCLUSION AND DISCUSSIONS

In this work, we give an example of DM engine using DM annihilation products as propulsion. The acceleration is proportional to the velocity, which makes the velocity increase exponentially with time in the non-relativistic region. The important points for the acceleration are how dense is the DM density and how large is the saturation region. The parameters of the spaceship also have great influence on the results. For example, the velocity will increase if S/M increases. We show that the (sub)halos can accelerate the spaceship to velocity $10^{-5}c \sim 10^{-3}c$ under the reasonable parameters of spaceship. Moreover, in case there is a central black hole in the halo, like galactic center, the core radius of DM can be large enough to accelerate the spaceship close to the speed of light.

We have used three assumptions in this work. First, we have assumed static DM for simplicity. But the DM particle may have velocity as large as $O(10^{-3}c)$. Once we know the velocity distribution of DM, it can be solved by programming the direction of the spaceship when speed is low. An analogue in our daily life is airplanes work well in both headwind and tailwind. Second, we have assumed the DM particles and the annihilation products can not pass through the wall of the engine. For the annihilation products, they may be SM fermions which have electric charges. Thus we can make them go into certain direction by the electromagnetic force. The most serious problem comes from DM which are weakly interacting with matter. Current direct searches of DM have given stringent bound on cross-section of DM and matter. It may be difficult using matter to build the containers for the DM, because the cross-section is very small. However, the dark sector may be as complex as our baryon world, for example the mirror world. Thus the material from dark sector may build the container, since the interactions between particles in dark sector can be large. Third, the annihilation process is assumed to happen immediately in the picture C. This is the second serious problem we should pay attention to. The annihilation speed takes the form, $A = \langle\sigma v\rangle \frac{\rho_{sat}^2}{2m_D^2}$. The $\langle\sigma v\rangle$ is taken to be the natural scale of the correct thermal relic, which is $3 \times 10^{-26} cm^3 s^{-1}$. One can show that $A = 2.2 \times 10^{-7} cm^{-3} s^{-1}$. However, the number density of the dark matter is $n_D = \frac{\rho_{sat}}{m_D} = 4 \times 10^9 cm^{-3}$. Thus, to make the annihilation process efficient, we have to compress the volume of the engine to raise the annihilation speed. Whether it can be achieved in the future is not clear. Nevertheless, the engine works in the vacuum where the baryonic matter is dilute, which means we do not need to worry about the pressure from the baryonic matter.

Sometimes, when looking at the N-body simulation pictures of DM, I think it may describe the future human transportation in some sense. In the picture, there are bright big points which stand for large dense halos, and the dim small points for small sparse halos. Interestingly, these halos have some common features with the cities on the Earth. The dense halos can accelerate the spaceship to higher speed which make it the important nodes for the transportation. However, the sparse halos can not accelerate the spaceship to very high speed, so the spaceship there would better go to the nearby dense halo to get higher speed if its destination is quite far from the sparse halos. Similarly, if we want to take international flight, we should go to the nearby big cities. The small cities usually only have flights to the nearby big cities, but no international flights. Thus we can understand the dense halos may be very important nodes in the future transportation, like the big cities on the Earth.

APPENDIX: DARK MATTER HALO AND SUBHALO PROFILES

Based on N-body simulations, the DM distribution can usually be parameterized as,

$$\rho(r) = \frac{\rho_s}{(r/r_s)^\gamma [1 + (r/r_s)^\alpha]^{(\beta-\gamma)/\alpha}}, \tag{A.1}$$

where ρ_s and r_s are the scale density and the scale radius parameters respectively. The parameters (α, β, γ) are $(1, 3, 1)$ for NFW profile. In this appendix, we briefly introduce how r_s and ρ_s are calculated. The two parameters can be determined once we know the (sub)halo mass M_v and the concentration parameter c_v which depends on the specific concentration model. The calculations are following the method in Ref.[4]. In the appendix of Ref.[8], we have shown how to determine the r_s in detail.

For the NFW profile, the scale radius r_s is

$$r_s^{nfw} = \frac{r_v(M_v)}{c_v(M_v)}. \tag{A.2}$$

where r_v is the virial radius of the subhalo which is often approximated as the radius within which the average density is greater, by a specific factor $\Delta = 200$, than the critical density of the Universe $\rho_c = 139 M_\odot kpc^{-3}$ ($M_\odot$ is mass of the Sun). Thus r_v can be expressed as $r_v = \left(\frac{M_v}{(4\pi/3)\Delta\rho_c}\right)^{1/3}$. The c_v is the concentration parameter of the subhalo which is determined by the subhalo mass M_v and concentration model. We use the same method as Ref.[4] which adopts two concentration models, which are ENS01[9] and B01[10]. In the Ref.[4], the c_v is fitted in a polynomial form as

$$\ln(c_v) = \sum_{i=0}^{4} C_i \times \left[\ln \frac{M_v}{M_\odot}\right]^i, \tag{A.3}$$

where $C_i = \{3.14, -0.018, -4.06 \times 10^{-4}, 0, 0\}$ and $\{4.34, -0.0384, -3.91 \times 10^{-4}, -2.2 \times 10^{-6}, -5.5 \times 10^{-7}\}$ for ENS01 and B01 model respectively.

The density scale ρ_s can be determined by the mass relation $\int \rho_s(r)\mathrm{d}V = M_v$. One can get the scale density ρ_s,

$$\rho_s^{nfw} = M_v/[4\pi r_s^3 A(c_v)], \tag{A.4}$$

where $A(c_v) \equiv In(1 + c_v) - c_v/(1 + c_v)$. In Fig.6, the scale radius r_s and the scale density ρ_s are plotted as a function of subhalo mass M_v to show how large and how dense the subhalos are. We can see the scale radius r_s is quite large which shows the acceleration is mostly done in the $r \ll r_s$ region of the (sub)halo.

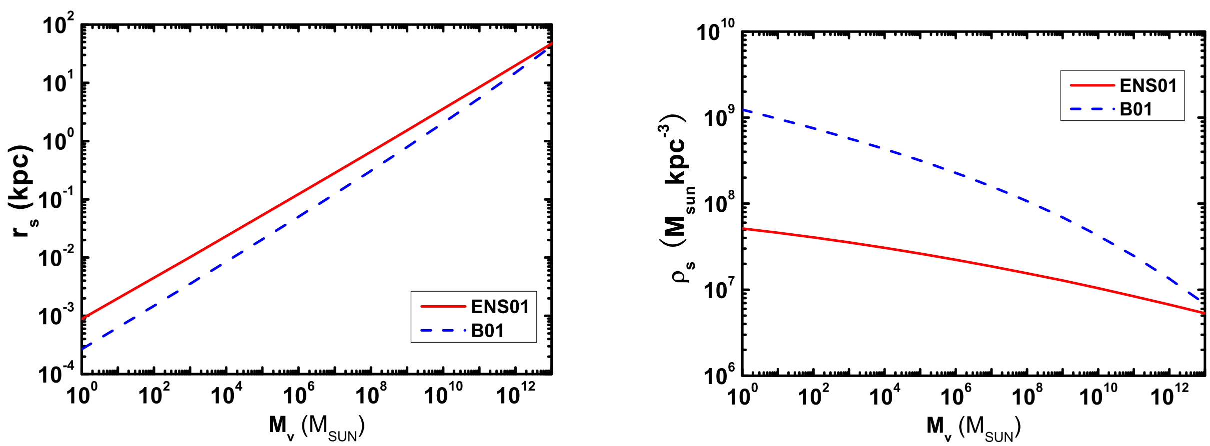

FIG. 6: The scale radius r_s and the scale density ρ_s as a function of (sub)halo mass M_v. This plot assumes the (sub)halos have NFW profile.

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